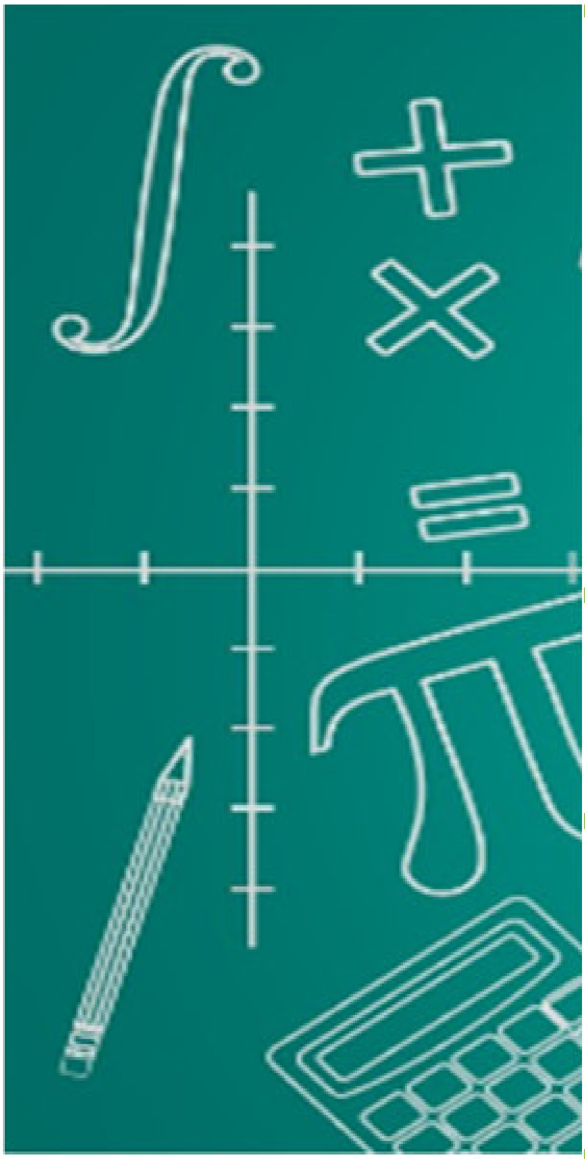

NAC MATHEMATICS LETTERS

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PREFACE

NAC Mathematics letters is the first departmental e-publication and it has been designed from the tutorial presentations of our present year odd semester Mathematics (Honours) students. Our students have prepared their write-up in taking help from different similar contents on website, like Wikipedia and also consulting different books available to them. Students also have taken suggestions from the faculties of the department. The department gratefully acknowledges the moral support received from our respected Principal, Dr. Jaydeep Sarangi, IQAC coordinator, Dr. Dhrubajyoti Banerjee and TCS , Dr. Avijit Paul. The department is also thankful to our beloved students for their kind cooperation to prepare this e-publication.

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Applications of differential equation in solving real life problems

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1. Introduction

Differential equations are important in mathematics and the sciences because they can be used to model a wide variety of real-world situations. A differential equation is defined as an equation containing the derivative of one or more dependent variables with respect to one or more independent variables. For example, $\frac{dy}{dx}x = y - 1$ where y is dependent variable and x is independent variable in the ordinary differential equation & $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 2z$ where z is the dependent variable and x and y are independent variables in the partial differential equation. The highest derivative which occurs in the equation is the order of ordinary differential equation. ODE for nth order in two variables can also be written as $F(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots\ldots, \frac{d^ny}{dx^n}) = 0$ .In real life problems, ordinary or partial differential equations are used to compute the flow of electricity, motion of an object, wave motion and to explain the concepts of thermodynamics.

2. Types of Differential Equation

a. Partial differential equation

- First-order partial differential equation
- Linear partial differential equation
- Quasi-linear partial differential equation
- Homogeneous partial differential equation

b. Ordinary differential equation

- Linear differential equation
- Non-linear differential equation
- Homogenous differential equation
- Non-homogenous differential equation

3. Uses of Differential Equation

Differential equations have many applications to get solutions of real life problems, such as in fields of

- Population dynamics & biological system
- Electronics & Circuit Design
- Option trading and economics
- Climatology and environmental analysis
- Vibration dynamics & Seismology
- Heat Transfer & Energy Balances
- Solid Mechanics & Motion

4. Discussions

In the study we have considered few applications where we can use the solutions of ordinary differential equations. The linear ODE can be solved by using the techniques of integrating factors, so that the equation becomes exact and can be solved by simple integration. The non-linear equations can be solved by using the techniques of method of separation of variables, method involving integrating factors and also by numerical techniques. The followings are few known areas of real life problems where the construction and finding of solutions of ordinary differential equation help us to understand the problem properly.

- **Population growth / decay model:**
  The differential equation describing the model of population growth and/ or decay of insects, bacteria or virus, animals and as well as human population at certain places and duration. is given by $\frac{dN(t)}{dt} = kN(t)$, where N(t) denotes population at time t and k is a constant of proportionality. The solution of the differential equation is given by $N(t) = C e^{-kt}$, where C is the integrating constant which can be determined if N(t) is given at time $t = t_0$.
- Drug distribution in human body:

To combat the infection to a human body appropriate dose of medicine is essential, because the amount of the drug in the human body decreases with time and the medicine must be given in multiple doses. The rate at which the level of the drug in a patient blood decays can be modelled by the decay equation

$$\frac{dy}{dt} = -ky$$

where k is a constant to be experimentally determined for each drug. If initially ie, at t=0,a patient is given an initial does y_p ,then the drug level y at any time t can be determined by the solution of above differential equation .ie,$y(t) = y_p e^{-kt}$ and is evaluated by using suitable integrating factor.

- **Newtons law of cooling:**

  It is a model that describes,mathematically the change in temperature of an object in a given environment. The law states that the rate of change (in time) of the temperature is proportional to the difference between the temperature T of the object and the temperature T_e of the environment surroundings the object

  $$\frac{dT}{dt} = -k(T - T_e) \cdot$$

  The solution of the above equation is given by $x = A e^{-kt}$,where $x = T - T_e$. It is assumed that at t = 0 the temperature $T = T_0$, then $T_0 - T_e = A e^0$ and hence $A = T_0 - T_e$. Thus the final expression for T(t) is given by $T(t)= T_e + (T_0 - T_e) e^{-kt}$The last expression explains how the temperature T of the object changes with time t.
- Efficacy skill measurment:
  The efficacy related problems can be calculated by using the first order ODE $\frac{dp}{dt} = k(M - P)$ which is a separable differential equation. The P(t) measures the performance of someone learning a skill after a training time t, M is the maximum level of performance and k is a positive constant.The solution of the above equation can be put as P(t) =k(M-P)+A,where A is the integrating constant.
- Survivability with Aids:

  The survival fraction S(t) is given by the ODE $\frac{dS(t)}{dt} = -k(S(t) - S_i)$ and it's solution is given by $S(t) =S_i+ (1- S_i) e^{-kt}$
- Game Apps development
  Theoretical models on game, building block concept and many applications are solve with the help of differential equations. Graphical interference of analysing data and creating browser are based on solution of partial differential equation by finite element method.

- RoboticIndustrialization
  Auto motion and robotic technologies for customized component, module and building prefabrication are based on differential equations.

Now a day, techniques of differential equations have proved that it is a significant part of applied and pure mathematics.It is also used in rocket science & modelling biological phenomenon, engineering science as well as weather forecasting and quantum mechanics.

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Applications of Group Theory

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Introduction

The Group theory is the branch of abstract-algebra that is incurred for studying and manipulating abstract concepts involving symmetry. Thus, Group theory is the tool to study symmetry. When we are dealing with an object that appears symmetric, group theory can help with the analysis. The term symmetric means to anything which stays invariant under some transformations.Group can be found in geometry, representing phenomena such as symmetry and certain types of transformations. Group theory has its applications in physics, chemistry, computer science, biology- genetic coding and other allied fields.

Definition

A non-empty set G is said to form a group with respect to a binary composition * , if

(i) G is closed under the composition,

(ii) * is associative,

(iii) there exists an element e in G such that a*e=e*a=e for all a in G. e is called the identity element

(iv) for each element in G, there exists an element a' in G such that a'*a=a*a'=e. a' is called the inverse element of a in G.

The group in denoted by the symbol (G,*).

Let's see few examples that will highlight the wide variety of groups and help to appreciate why it took a while for mathematicians to recognize that they were using the same tool in completely different settings.

Example 1:

Consider the set of integers: Z {………-3,-2,-1,0,1,2,3,….}
and the single binary operator: "+". Now investigate the properties of Group w.r.t. "+" .
Closure property: For integer values of x,y, x+y is also an integer.

Associative property: For integer values of x,y,z , (x+y)+z = x+(y+z)
Identity / Zero element: For integer values of x and 0, x+0 = 0+x = x. "0" is the identity element.
Inverse element: For integer x there exist integer (-x) , such that x+(-x)= 0.
So, (Z,+) is a group.

Example 2:

Let's imagine that we live on an alien planet where each day has 7 hours.

CLOCK ARITHMETIC
Day Length: 7 hours
Clock Numbers:0,1,2.3.4.5.6
Clock Arithmetic:
3+5=1(Move Clockwise)
4+3=0
1-3=5(Move Counter Clockwise)
So,

MODULAR ARITHMETIC
7 Hours⇒Integers (mod 7)
Here "mod" is for "modular"

CLOCK ARITHMETIC → MODULAR ARITHMETIC

Importance of Group Theory

i. Group theory actually has a huge number of applications in the real world. Group theory has many applications in Medical imaging, Computer vision etc.
ii. When we want to buy something through online, we are to send our credit card details or equivalent securely and that is done via encryption without telling anyone how to decode it. This is actually possible by exploiting the natural structure of groups.
iii. Group theory finds an application in card tricks. Suppose we place a card in the middle of a deck, how can we shuffle the deck so that it always ends up as the top card? All these

types of questions and more can be answered with group theory. We can similarly find a method of solving Rubik's cubes by considering the possible movements as the actions of groups.

iv. A group is called cyclic if there is an element that generates the entire set by repeatedly applying an operation. A cyclic group could be a pattern found in nature.For example, in a snowflake or in a geometric pattern.Cyclic group used in Cryptology and Number Theory and other applications including music and chaos theory.

Natural objects that are Cyclic:

Urchin

Flower

v. Rotation are one of the common applications of cyclic group.We can draw a square moving 90 degrees 4 times.For a polygon with n sides,we can divide 360/n to determine how many degrees each rotation will be to return to the original position.The rotation of circle is not cyclic.

An object with rotational symmetry is also known in biological contexts as radial symmetry.

<u>Rotational Symmetry</u>

vi. Method of ringing,known as scientific ringing,is the practice of of ringing the series of bells as a series of permutation.A permutation f: {1,2,…,n} → {1,2,…,n},where the domain numbers represent position and range numbers represent the bells.f(1)would ring the bell first and bell f(n)last.The number of bells n has n! possible changes.No bell must be run twice in a row.

A common permutation pattern for four bells is the Plain Bob Minimum Permutations. The Plain Bob pattern switches the first two bells then the second set of bells, they would start the bell ringing with 1234.The first bell would go to the second position and third would go to the forth position. Therefore, the next bell combination would be 2143.The next bell switch would be the two middle bells. The bell 2143 would turn to 2413.The bell ringers would repeat this pattern of switching the first two and second two followed by switching the middle until about 1/3 we switch the middle two,we would get back 1234.Therefore the bell ringers figured out to switch the last two bells every 8 combinations. Then after 24 moves(4!) we get back to the bell combination of 1234. Thus, we can say that the bell ringing pattern is cyclic.

vii. Biological application of group theory-Cell cycle and multinucleated cell

Cell cycle is the natural application of group theory because of the cyclic symmetry governing the process back to G1. The only reasonable approach for casting the cell cycle into group theory is to use the symmetries of a square.

Writing the rotation operations for the cell cycle as permutations we get:

$$R_0 = \begin{pmatrix} G_1 & S & G_2 & M \\ G_1 & S & G_2 & M \end{pmatrix} \quad R_{90} = \begin{pmatrix} G_1 & S & G_2 & M \\ S & G_2 & M & G_1 \end{pmatrix}$$

$$R_{180} = \begin{pmatrix} G_1 & S & G_2 & M \\ G_2 & M & G_1 & S \end{pmatrix} \quad R_{270} = \begin{pmatrix} G_1 & S & G_2 & M \\ M & G_1 & S & G_2 \end{pmatrix}$$

Group table for cell cycle:

	G_1	**S**	**G_2**	**M**
G_1	G_1	S	G_2	M
S	S	G_2	M	G_1
G_2	G_2	M	G_1	S
M	M	G_1	S	G_2

The cell cycle group table suggests exploring the group operations of some actual physical manipulation. Experiments on transferring nuclei from one cell into another to produce cells with multiple nuclei.Here we examine by means of group table, the converge state for those binucleated cells.

Group table for the converged state of binucleated cells:

	G_1	**S**	**G_2**	**M**
G_1	G_1	S	G_1/G_2	M
S	S	S	S	M
G_2	G_1/G_2	S	G_2	M
M	M	M	M	M

The multiplication table shows that the set$\{G_1,G_2,S,M\}$ carries the structure of a groupoid. similar considerations apply if we fuse cells of different type.

Discussion

Group theory is the natural language to describe the symmetries of a physical system. Outcomes, obtained from group theory could only be useful if and only if an individual understands them well enough to consider and deploy them up and provide users with a few basis insights as earlier as possible.

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Congruence and its applications

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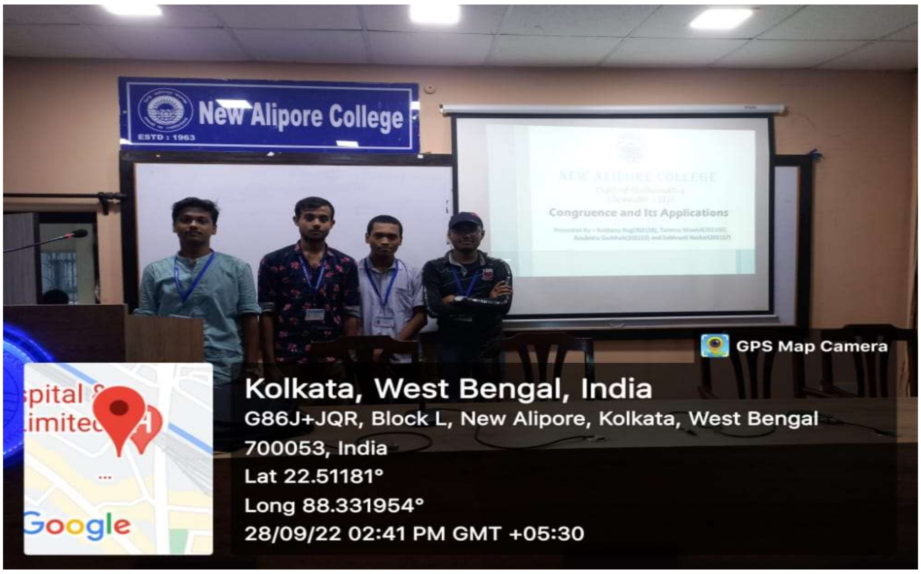

Introduction

The idea of congruences was developed by famous German mathematician Karl Friedrich Gauss (1777-1855). A new branch of mathematics "Number theory" was thus created. Gauss described "Mathematics is the Queen of the sciences and the theory of numbers is the Queen of Mathematics." The theory of congruences is an approach to test the divisibility of integers on using arithmetic of remainders. Gauss introduced this powerful theory with an outstanding notation of congruence in 1801, when he was only 24. According to him "if a number n measures the difference between two numbers a and b, then a and b are said to be congruent with respect to n; if not, incongruent."

Definition

Let 'm' be a fixed positive integer. Two integers a and b are said to be congruent modulo m, denoted by $a \equiv b \pmod{m}$, if a-b is divisible by m (i.e., m divides a-b or a-b = km for some integer k)

As the notation used above, we sometimes say that a is congruent to b modulo m. If a - b is not divisible by m (i.e., m does not divide a - b), we say that a is not congruent to b modulo m.

Few Illustrations of congruence taking some integers

Illustration-1: Take m = 7, a positive integer. We have $20 \equiv 6 \pmod{7}$, $-1 \equiv 6 \pmod{7}$, $4 \equiv -3 \pmod{7}$, as 20-6=14, -1-6=-7 and 4+3=7 are all divisible by 7. But 10 is not congruent to 2 modulo 7, as 10-2 = 8 is not divisible by 7.

Illustration-2: We have $18 = 2 \times 7 + 4$ and $-17 = (-3) \times 7 + 4$

Thus 18 and -17 leave the same reminder 4, when divided by 7.Clearly, $18 \equiv -17 \pmod{7}$.

Some Important Properties on Congruences

A. If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ where a, b, c, d are integers and m is a positive integer, then

1. $a \pm c \equiv b \pm d \pmod{m}$
2. $ac \equiv bd \pmod{m}$
3. $ax + cy \equiv bx + dy \pmod{m}$, x and y are integers.

Example: we have, $36 \equiv 8 \pmod{7}$ and $-2 \equiv 5 \pmod{7}$

Here, (i) $(36 \pm 2) \equiv (8 \pm 5) \pmod{7}$, which is true

(ii) $36x(-2) \equiv 8x5 \pmod{7}$ which is true

(iii) $(36 \times 3) + (-2 \times 13) \equiv (8 \times 3) + (5 \times 13) \pmod{7}$ which is true

B. If $a \equiv b \pmod{m}$, c be any integer and n be a positive integer, then

1. $a \pm c \equiv b \pm c \pmod{m}$
2. $ac \equiv bc \pmod{m}$
3. $a^n \equiv b^n \pmod{m}$

Example: we have, $36 \equiv 8 \pmod{7}$ and $-2 \equiv 5 \pmod{7}$

Here, (i) $(36 \pm 2) \equiv (8 \pm 2) \pmod{7}$, which is true

(ii) $36 \times 3 \equiv 8 \times 3 \pmod{7}$ which is true

(iii) $36^2 \equiv 8^2$ (mod 7) which is true

However, $ac \equiv bc$ (mod m) does not necessarily imply $a \equiv b$ (mod m). We notice that $2.8 \equiv 2.5$ (mod 6), but 8 is not congruent to 5 modulo 6.

Again, $a^n \equiv b^n$ (mod m) also does not necessarily imply $a \equiv b$ (mod m). For example, $(-2)^3=3^3$ (mod 7), but -2 is not congruent to 3 modulo 7.

C. Congruence is an Equivalence relation on $\mathbb{Z}$

Some Problems on Congruence

Problem 1:

Find all integers k, $1 \leq k \leq 100$ such that $k \equiv 10$ (mod 17).

Solution: Given: $k \equiv 10$ (mod 17)

Therefore, there exists integer p such that

$p.17=k-10 \Rightarrow k = 10 + 17p$

$\Rightarrow k = 10, 27, 44, 6, 78, 95$ (for p = 0; 1, 2, 3 4 and 5 respectively)

Here p cannot take any negative integer and any integer ≥ 6, as $k \geq 1$

Problem 2:

Find the remainder when 7^{20} is divided by 4.

Solution: We know $7 \equiv -1$ (mod 4)

$\Rightarrow 7^{20} \equiv (-1)^{20}$ (mod 4) [as 7-(-1) = 8 is divisible by 4]

$\Rightarrow 7^{20} \equiv 1$(mod 4)

Thus, the remainder when 7^{20} is divided by 4 is 1.

Application of Congruences

Congruence has wide applications in real life problems. The main fields where congruence is applied widely are as follows:

I. To determine a valid **ISBN** (both ISBN-10 and ISBN-13).

II. To determine a valid **ISSN**.

III. To determine a valid **UPC**.

IV. To determine credit card numbers.

I. **ISBN:**

ISBN stands for International Standard Book Number. The ISBN is a unique number to identify each book published globally. ISBN can be of 10 digits (ISBN-10) or 13 digits (ISBN-13) . The last digit of both the ISBN (ISBN-10 or ISBN-13) is called the **Check Digit** which is used to protect any possible error from imperfect transcription of that particular ISBN.

- **Computing the Check Digit**

  - **ISBN-10:**ISBN-10 consists 10 alphanumeric digits therefore ISBN of any book having ISBN-10 can be expressed as $a_1a_2a_3a_4a_5a_6a_7a_8a_9c$ where each of a_i(i=1,2,…,9) are numbers from the set{0,1,2,3,…,9}. c is the check digit and is a number also from the set {0,1,2,…,9}.

    Formula to compute the **Check Digit:**

    $$1.a_1 + 2.a_2 + 3.a_3 + 4.a_4 + 5.a_5 + 6.a_6 + 7.a_7 + 8.a_8 + 9.a_9 + 10.c \equiv 0 \pmod{11} \quad (2)$$

Illustration:

Consider the book entitled 'Complex Variables' written by M.R. Spiegel et al., under Schaum's Oullines and published by Mc Graw-Hill. The ISBN-10 of that book is given by 0-07-008538-2. Using congruence, we will check whether this ISBN is valid or not.

Since the above ISBN must satisfy the relation (2), then we have

1.0 + 2.0 + 3.7 + 4.0 + 5.0 + 6.8 + 7.5 + 8.3 + 9.8 = c(mod 11)

i.e., 21 + 48 + 35 + 24 + 72 = c (mod 11)

i.e., c (mod 11) = - 1 + 4 + 2 + 2 – 5

i.e., c (mod 11) = 2

Therefore, the check digit is 2 which is exactly same as the above-mentioned ISBN. Hereby, it is a valid ISBN.

  - **ISBN – 13:** ISBN-13 consists 13 digits therefore ISBN of any book having ISBN-13 can be expressed as $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12}c$ where each of a_i(i=1,2,…,12) are numbers from the set {0,1,2,3,…,9} and c is the check digit and is also a number from the set {0,1,2,…,9}.
    Formula to compute the **Check Digit:**

    $$1 \times a_1 + 3 \times a_2 + 1 \times a_3 + 3 \times a_4 + 1 \times a_5 + 3 \times a_6 + 1 \times a_7 + 3 \times a_8 + 1 \times a_9 + 3 \times a_{10} + 1 \times a_{11} + 3 \times a_{12} + c \equiv 0 \pmod{10}$$

II. **ISSN:**

ISSN is the International Standard Serial Number and this number is also another unique number used to identify the journals. A valid ISSN contains 8 numbers from the set {0,1,2…,9}. Let the ISSN of a journal be $a_1a_2a_3a_4a_5a_6a_7c$ where c is the check digit.

Formula to compute the **check digit:**

$$\mathbf{c + 2 \times a_7 + 3 \times a_6 + 4 \times a_5 + 5 \times a_4 + 6 \times a_3 + 7 \times a_2 + 8 \times a_1 \equiv 0 \pmod{11}}$$

III. **UPC:**

UPC or Universal Product Code is a 13 digit unique code and this code is used to identity a product of retail store or even from grocery. Every valid UPC contains 13 numerical characters including the last character as the check digit. The rule for determining a valid UPC is same as ISBN-13. Let us agree to write a valid UPC as $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12}c$

Where $0 \leq a_i \leq 9 \ \forall \ i = 1, 2,\ldots, 12$ and $0 \leq c \leq 9$, where c is the check digit.

Formula to Compute the **check digit:**

$1 \times a_1 + 3 \times a_2 + 1 \times a_3 + 3 \times a_4 + 1 \times a_5 + 3 \times a_6 + 1 \times a_7 + 3 \times a_8 + 1 \times a_9 + 3 \times a_{10} + 1 \times a_{11} + 3 \times a_{12} \equiv (10\text{-}c) \pmod{10}$

when the last digit of the sum given in left hand side in not equal to zero

Illustration:

Consider the UPC of a product as 8906032713739. We will check whether this UPC is valid or not.

For the present UPC we have $\sum_{i=1,\, i \text{ is odd}}^{12} a_i + 3 \sum_{i=1,\, i \text{ is even}}^{12} a_i = (8 + 0 + 0 + 2 + 1 + 7) + 3(9 + 6 + 3 + 7 + 3 + 3) = 18 + 3 \times 31 = 111 \equiv 1 \pmod{10}$

Therefore, by the formula we get the check digit c will be 10-1 = 9. Hence, the above UPC is valid.

IV. **Credit Card:**

Every credit card contains 16 non-negative integers as digits where the last digit is the check digit which indicates the validity or authentication of that credit card. A credit card with 16 digits is expressed as $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_{10}a_{11}a_{12}a_{13}a_{14}a_{15}c$ where $0 \leq a_i \leq 9$ for i = 1,2,…,15 and c is a check digit.

Formula to compute the **check digit:**

$S_1 = \sum_{i=2,\, i \text{ is even}}^{16} a_i = (a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12} + a_{14} + c)$

Next we will multiply every digit at odd position of the above credit card with two and obtain a sequence of 8 numbers as follows : $2a_1, 2a_3, 2a_5, 2a_7, 2a_9, 2a_{11}, 2a_{13}, 2a_{15}$, next we add all digits of the above sequence and say it as S_2. If one or more member of the sequence have to two digits then we need to add those two digits to obtain S_2. Now the check digit can be obtained from the following congruence which is

$\mathbf{S_1 + S_2 \equiv 0 \pmod{10}}$

i.e., $\mathbf{c + \{S_2 + (a_2 + a_4 + a_6 + a_8 + a_{10} + a_{12} + a_{14})\} \equiv 0 \pmod{10}}$

Illustration:

Consider the first 15 digits of an identification number of a credit card as a 4550 3891 6078 001. Using congruence find the 16th digit of this credit card.

Here, $S_1 = 5 + 0 + 8 + 1 + 0 + 8 + 0 + c = 22 + c$

Next, we multiply each odd position digit of the above card number by 2 to obtain a sequence of 8 numbers which are as follows

$4 \times 2 = 8$, $5 \times 2 = 10$, $3 \times 2 = 6$, $9 \times 2 = 18$, $6 \times 2 = 12$, $7 \times 2 = 14$, $0 \times 2 = 0$, $1 \times 2 = 2$.

Now, the sum of the digits of the above sequence is,

$8 + (1 + 0) + (1 + 8) + 6 + (1 + 2) + (1 + 4) + 0 + 2 = 34 = S_2$(say)

Now, in order to above card number becomes as an authentic card number it should satisfy the following congruence: $S_1 + S_2 \equiv 0 \pmod{10}$

$\Longrightarrow (22 + c) + 34 \equiv 0 \pmod{10}$ $\Longrightarrow c = 4$

Thus, the required check digit should be 4 and hence this credit card number is **4550 3891 6078 0014**

Discussion

Congruences are important and useful tool for the study of divisibility and related real-life problems.

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Applications of Assignment and Travelling Salesman Problem

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Introduction

The assignment problem is a fundamental combinatorial optimization problem. In its most general form, the problem is as follows: The problem instance has a number of agents and a number of tasks. Any agent can be assigned to perform any task, incurring some cost that may vary depending on the agent-task assignment. The Travelling salesman problem is very similar to the assignment problem except that in the former, there are additional restrictions that a salesman starts from his city, visits each city once and returns to his home city, so that the total distance (cost or time) is minimum.

Description of an Assignment Problem

Suppose a company has one salesman and the company wants to sell its product in a city. How would they employ the salesman? Company has no alternatives but to engage that only salesman in the city for selling the products. Again, suppose there are two salesmen in the company and two cities with different potentialities where the company wants to sell its products. Which salesman should engage to which city according to the ability of the salesman and potentiality of the city for maximizing profit? Similarly, if there are n salesmen and n cities which salesman should be assigned to which city to ensure maximum profit? While answering the above questions we have to think about the interest of the company. So, we have to find such an assignment by which the company gets maximum profit by selling its products. Such type of problem is known as Assignment problem. Assignment problem is to be considered as a special case of transportation problem where each supply is 1 and each demand is 1. In this case every supplier will be assigned one destination and every destination will have one supplier.

Mathematical formulation of an Assignment Problem:

Let x_{ij} be a variable defined by:

x_{ij} = 1 if i^{th}origin is assigned to j^{th}destination.

x_{ij} = 0 if i^{th} origin is not assigned to j^{th} destination.

Now the assignment problem is

Optimize (Minimize or Maximize)

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i = 1$, i=1,2,……..n

$$\sum_{i=1}^{n} x_{ij} = b_j = 1, \text{ j=1,2,……..n}$$

From the above discussions it is clear that the problem before us is to select n cells in a nxn transportation table, only one cell in each row and each column, such that the sum of the corresponding costs (or profits) be Minimum (or Maximum). Obviously, the solution obtained is a degenerate solution. Thus, an assignment problem is a particular type of L.P.P. with Basic Feasible Solution always degenerate.

Method of solution

To find an optimal solution of an assignment problem, we follow a simple algorithm called **Hungarian method.** The Hungarian mathematician **D. Konig**developed an efficient method for finding an optimal solution. The procedure is as mentioned below:

Step 1: Prepare a cost matrix. If the cost matrix is not a square matrix, then add a dummy rowor dummy column with zero cost elements.

Step 2: Subtract the minimum element in each row from all the elements of the respective rows. Therefore, there will be at least one zero in each row of this new matrix which is called the first reduced cost matrix.

Step 3: Further subtract the smallest cost element in each column from all elements of the respective columns. As a result, there would be at least one zero in each row and column of the second reduced cost matrix.

Step 4: Determine an optimum assignment as follows:

i. Starting with first row of the second reduced matrix, examine the rows of this matrix which contains only one zero in it. Mark this zero within the circle and cross out the columns containing the assigned zeros. Proceed in this manner until all the rows have been examined. If there are more than one zero in any row, then do not consider that row and pass on to the next row.
ii. Start from the first column and examine all the uncovered column to find the columns containing exactly one remaining zero. Mark this zero within the circle as an assignment will be made there. Cross out the rows containing this assigned zero.

Repeat steps (i) and (ii) until all zeros are either assigned or crossed out.

Step 5: If the minimum number of lines required to cover all zeros is equal to the order of the cost matrix, then the assignment made is step-4 is the optimal solution. Otherwise go to the next step.

Step 6: Revise the cost matrix as follows:

Select the smallest element among the uncrossed elements. Then subtract this element from all the uncrossed elements and add the same at the point of intersection of two crossed out lines whereas the other elements crossed by the lines remain unchanged.

Step 7: Go to step 4 and repeat the procedure till an optimum solution is attained.

The value of the objective function is obtained by adding the original values of the assignment matrix in the assigned cell.

Example:

	a	b	c	d	e
A	11	6	14	16	17
B	7	13	22	7	10
C	10	7	2	2	2
D	4	10	8	6	11
E	13	15	16	10	18

(1)

A car hire company has one car in each of the five depots a, b, c, d and e. A customer in each of the five towns A, B, C, D and E requires a car. The distance (in km) between the depots (origins) and the towns (destinations) where the customers are, given by the distance matrix.How should the cars be assigned to the customers so as to minimize thedistancetravelled?

Solution:

	a	b	c	d	e
A	5	0	8	10	11
B	0	6	15	0	3
C	8	5	0	0	0
D	0	6	4	2	7
E	3	5	6	0	8

(2)

	a	b	c	d	e
A	8	0	8	13	11
B	0	3	12	0	0
C	11	5	0	3	0
D	0	3	1	2	4
E	3	2	3	0	5

(3)

Hence, A → b, B → e, C →c, D → a, E → d

Therefrom, from table (1), Minimum distance = 6+10+2+4+10 = 32 km. (Answer)

Types of Assignment Problems

❖ Maximization Assignment Problem:

If an assignment problem deals with maximization of the objective function then obviously the matrix considered is a profit matrix. Such problem may be solved by first reducing it to a problem of minimization and then applying the usual technique of assignment algorithmic. This can be done by subtracting all the elements of the profit matrix from the largest elementof the matrix.

❖ Restricted Assignment:

During assignment, some restrictions are being adopted considering from the economical, commercial or physical points of view. Such types of assignment problems are called restricted assignment problems. The restriction is generally indicated by putting cross (x) or '-' in the corresponding cell. The difficulty for solving such type of problem can be overcome by assigning a very large value M or ∞ in those restricted places.

❖ Unbalanced Assignment Problem:

An assignment problem is said to be unbalanced if the number of Assignee and the number of Assignment be not same and the cost or profit matrix be not a square matrix. Here either the number of assignee is greater than the number of assignment and vice-versa. But the problem can also be solved by converting the matrix (cost or profit) into a square matrix by introducing either fake assignee or assignment whichever is necessary and assuming all costs components zero corresponding to that assignee or assignment

Applications of assignment problem

i. To minimize the total cost or time required to complete task.
ii. To assign a best job to best person.
iii. To assign vehicles to the routs.
iv. To assign sales representative to the sales territories.
v. In minimizing the time of arrival and departure of airlines.

Method of Solving Travelling Salesman problem

As the travelling salesman problem is very similar to assignment problem, so Hungarian method can be used to solve T.S.P. But the optimal solution must obey additional constraint, say the salesman visits the cities in such a way that he takes the step to visit the next destination from the currently visited city and finally return to the starting city. If the solution dose not satisfy this additional restriction, then after solving the problem by assignment technique we use the method of enumeration. During enumeration if non-zero cells are required to complete the programme, only the lowest non-zero cells should be considered to minimize the number of enumerations.

Example

A medical representative has to visit five stations A, B, C, D and E. He does not want to visit any station twice before completing his tour of all stations and wishes to return to the starting station. Cost of going from one station to another is given below. Determine the optimal route and the minimum expenditure to be done by the representative.

	A	B	C	D	E
A	∞	12	15	10	8
B	8	∞	15	12	8
C	9	11	∞	15	11
D	7	12	19	∞	11
E	9	12	16	10	∞

(1)

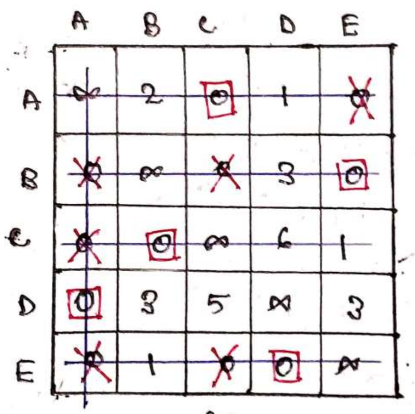
Solution:

Step 1:Subtracting the lowest element of each row from all other elements of the corresponding row we get the new table.

	A	B	C	D	E
A	∞	4	7	2	0
B	0	∞	7	4	0
C	0	2	∞	7	2
D	07	5	12	∞	4
E	0	3	7	1	∞

(2)

Step 2:Now subtract the lowest element of each column from the all other elements of the corresponding column we get the following table.



	A	B	C	D	E
A	∞	2	0	1	0
B	0	∞	0	3	0
C	0	0	∞	6	1
D	0	3	5	∞	3
E	0	1	0	0	∞

(3)

All zeros can only be covered by minimum five lines which are shown in the table.

Now from the table, construct the table containing only zeros at their proper position after removing all zeros at the points of intersection of the lines. Selection of only one zero in each row and each column is not unique. One such selection is given above. The restricted optimal assignment is A->C, B->E, C->B, D->A, E->D and the min cost is 15+8+11 + 7+10 = 49 units.

Now if we arrange to route in the manner A->C->B->E->D->A then it follows the route of the travelling salesman problem and hence the optimal cost of the travelling salesman problem is 49 units and one route is A->C->B->E->D->A. (answer)

Discussion

The Assignment problem is a special case of transportation problem. Unlike a transportation problem, in an assignment problem, number of facilities (sources) is equal to number of jobs (destinations). But the Travelling Salesman Problem (TSP) is the challenge of finding the shortest yet most efficient route for a person to take given a list of specific destinations. It is a well-known algorithmic problem in the fields of computer science and operation of research.

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Applications of Linear Programming Problem

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Introduction

Linear Programming Problem or L.P.P. is a mathematical technique to optimize the consumption of resources. It is thus a problem that is concerned with finding the optimal value of a linear function. The optimal value can be either maximum value or minimum value. Here, the given linear function is considered as an objective function. The variables in the objective function, are the unknown quantities that are expected to be estimated subject to the validation of the constraints, as an output of the LPP solution. The cconstraints are nothing but the basic restrictions or limitations of resources consumed man, money, machine, material and even some data from market research also. The techniques of LPP can be applied to solve various types of problems such as Manufacturing problems, Diet problems, Transportation problems, Optimal Assignment problems, Staff Management problems and the like.

Mathematical formulation of L. P. P.

Linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality of constraints. Thus, Linear programming are problems that can be expressed in canonical form as

Find a vector X,
that Maximizes or Minimizes $C^T X$
subject to $AX \le b$ or $AX \ge b$
and $X \ge 0$

Applications of L. P. P.

1. Staff management problemof Nurses in a Hospital:

PERIOD	CLOCK TIME (24 HRS. A DAY)	MINIMUM OF NURSES REQUIRED
1	6 AM -- 10 AM	60
2	10 AM -- 2 PM	70
3	2 PM – 6 PM	60
4	6 PM –10 PM	50
5	10 PM – 2 AM	20
6	2 AM –6 AM	30

Nurses report to the hospital wards at the beginning of each period and work for 8 consecutive hours. The hospital wants to determine the minimum number of Nurses so that there may be sufficient number of Nurses available for each period.

Let A be Number of Nurses reporting at beginning of Period 1

Let B be Number of Nurses reporting at beginning of Period 2

Let C be Number of Nurses reporting at beginning of Period 3

Let D be Number of Nurses reporting at beginning of Period 4

Let E be Number of Nurses reporting at beginning of Period 5

Let F be Number of Nurses reporting at beginning of Period 6

and Let Z be the total Number of required Nurses.

Since same Nurses can't work for more than two consecutive periods, So

A Nurses work for the periods 1 and 2

BNurses work for the periods 2 and 3

CNurses work for the periods 3 and 4

DNurses work for the periods 4 and 5

ENurses work for the periods 5 and 6

FNurses work for the periods 6 and 1

Formulation of the Problem:

Minimize Z = A + B + C+ D + E + F

Subjected to, $F + A \geq 60$
$A + B \geq 70$
$B + C \geq 60$
$C + D \geq 50$
$D + E \geq 20$
$E + F \geq 30$
$A, B, C, D, E, F \geq 0$

2. FEED MIX problem:

A doctor advice a patient to take at least 150 calories out of two kinds of food-milk and meat and also advices him not to take more than 18 unit of fat daily. Relevant information is given in following table:

FOOD TYPE	CALOROIE (Kcal)	UNIT OF FAT	COST
MILK (PER LITRE)	120	15	Rs. 45
MEAT (PER KG.)	300	30	Rs. 600

Formulate the problem as a LPP in order to have minimum cost diet.

Table for formulation:

FOOD TYPE	CALOROIE (Kcal)	UNIT OF FAT	COST
MILK (PER LITRE)	120	15	Rs. 45
MEAT (PER KG.)	300	30	Rs. 600
REQUIREMENT	150	18	

Variables: X ltrs. of Milk and Y kg. of Meat

Formulation of the Problem:

Minimize Cost $Z = 45X+600Y$
Subject to : Daily Kcal Intake $120X+300Y \geq 150$
Daily Fat Intake $15X+30Y \leq 18$
Amount of Milk & Meat $X,Y \geq 0$

Advantages of LPP:

1. Optimum use of productive resources.

2. Improves the quality of decisions.
3. Provides possible and practical solutions.
4. Highlight the bottlenecks in the production processes.
5. Helps in re-evaluation of a basic plan for changing conditions.

Limitations of LPP:

1. It is not easy to define a specific objective function.
2. It is based on the assumption of constant returns.
3. It assumes perfect competition in product and factor markets.
4. Presents trial and error method.
5. The solutions obtained in LPP may not be integers all the time.

Assumptions:

1. Conditions of Certainty: Numbers in the objective and constraints are known with certainty and do change during the period being studied.
2. Linearity or Proportionality: We also assume that proportionality exits in the objective and Constraints.
3. Additively & Divisibility: It means that total of all activities quails the sum of each individual activity. We make the divisibility assumption that solution need to be integers.

4. Non-negative variable: In LP problems we assume that all answers or variables are non-negative.

5. Finiteness & Optimality: An optimal solution cannot compute in the situation where there is infinite number of alternative activities and resources restriction. In linear programming problems of maximum profit solution or minimum cots solution always occurs at a corner point of the set of the feasible solution.

Discussion

Linear programming provides a method to optimize operations within certain constraints. It is used to make processes more efficient and cost-effective. Some other areas of applications for LPP include Food and Agriculture, Engineering, Transportation, Manufacturing and Energy.

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[1] J. G. Chakraborty and P. R. Ghosh, Linear Programming and Game Theory, Moulik Library, Kolkata

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Linear Programming Problems

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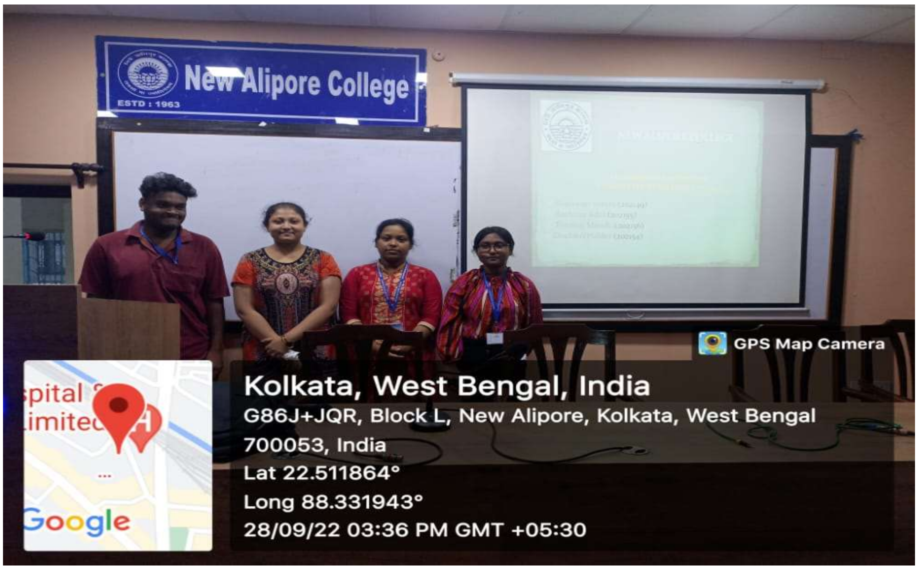


Introduction

Linear programming is a widely used mathematical technique to determine the optimum allocationof resources among competing demands. Resources typically include raw materials, machinery, time, money and space.The technique is very powerful and found especially useful because of its application to many different types of real business problems in areas like finance, production, distribution of personnel, marketing and many more areas of management.As its name implies, the linear programming model consist of linear constraints, which means that the variables in a model have a proportionate relationship.

Fundamentals of LPP model

a. **Objective function**: A linear function of the objective which either to maximize or minimize, like maximize profit, sales, production etc. and minimize cost, loss, energy, consumption, wastage etc.
b. **Decision variable:** The variables which are changeable & going to impact the decision function, like the profit function is affected by both sales and price, now which one of these two is changeable, will be our decision variable**.**

c. **Constraints:** Any kind of limitation or scarcity explained through a function like limitations of raw materials, time, funds, equipment etc. non-negative constraints will also be there which will remain non-negative all the time.

Formulation of a Linear ProgrammingProblem

a. First, recognize the decision variables and assign symbols to them like x,y,z& so on . Now they are the quantities, we wish to find out.
b. Next, express all the constraints in terms of inequalities in relation to the decision variables.
c. Now, formulate the objective function in terms of the decision variables.
d. Finally, add the non-negativity condition to the constraints.

General Linear Programming Model

A general representation of linear programming model is given as follows:
Maximize or Minimize, $Z = p_1x_1 + p_2x_2 + p_3x_3 + ... + p_nx_n$

Subject to constraints,
$w_{11}x_1 + w_{12}x_2 + ... + w_{1n}x_n \leq or = or \geq b_1$
$w_{21}x_1 + w_{22}x_2 + ... + w_{2n}x_n \leq or = or \geq b_2$

$w_{m1}x_1 + w_{m2}x_2 + ... + w_{mn}x_n \leq or = or \geq b_m$
Non-negativity constraint,
x_i ≥0(where i= 1,2,3………n)

- **Example 1:**

A company manufactures two types of boxes, corrugated and ordinary cartons.The boxes undergo two major process : cutting and pinning operations. The profit per unit are Rs. 6 and Rs. 4 respectively. Each corrugated box requires 2 minutes for cutting and 3 minutes for pinning operation, whereas each carton box requires 2 minutes for cutting and 1 minutes for pinning.The available operating time is 120 minutes and 60 minutes are cutting and pinning machines.The manager has to determine the optimum quantities to be manufacture the two boxes t maximize the profits.

Decision variables: Completely describe the decisions to be made (in the present case , by the Manager). Manager must decide how many corrugated and ordinary cartons should be manufactured each week. With this in mind, he has to define:

x_1 be the number of corrugated boxes to be manufactured.

x_2 be the number of carton boxes to be manufactured

Objective function: **It**is the function of the decision variables that the decision maker wants to maximize or minimize. Manager can concentrate on maximizing the total weekly profit(Z).

Here profit equals to (Weekly revenues)-(Raw material purchase cost)-(Other variable costs).

Hence Manager's objective function is:

Maximize Z = $6x_1 + 4x_2$

Constraints: It shows the restrictions on the values of the decision variables. Without constraints manager could make a large profit by choosing decision variables to be very large. Here there are three constraints:

- available machine-hours for each machine
- time consumed by each product

Sign restrictions are added if the decision variables can only assume non-negative values

(manager cannot use negative number machine and time never negative number)

Thus, the LPP becomes

Max Z = $6x_1 + 4x_2$ (the objective function)

Such that $2x_1 + 3x_2 \leq 120$ (cutting time constraint)

$2x_1 + x_2 \leq 60$ (pinning constraint)

$x_1, x_2 \geq 0$ (sign restrictions)

A value of (x_1, x_2) is in the feasible region if it satisfies all the constraints and sign restrictions .This type of linear programming can be solve by two methods

1) Graphical method

2) Simplex algorithm method

Solution through Graphical Method

Step 1: First, Convert the inequality constraints as equations and find co-ordinates of the line.

Step 2: Next, Plot the lines on the graph.

Step 3: We Obtain the feasible zone.

Step 4: Next, Find the co-ordinates of the objective functions and plot it on the graph

representing itwith a dotted line.

Step 5: Now, we locate the solution point.

Step 6: Finally, investigate different types of solution:

i. If the solution point is a single point on the line, take the corresponding values of x_1 and x_2.
ii. If the solution point lies at the intersection of two equations, then solve for x_1 and x_2 using the two equations.
iii. If the solution appears as a small line, then a multiple solution exists.
iv. If the solution has no confirmed boundary, the solution is said to be an unbounded solution.

- **Example 2:**

A furniture company produces inexpensive tables and chairs. The production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labour hours in the painting department. Each table takes 4 hours of carpentry and 2 hours in the painting department. Each chair requires 3hours of carpentry and 1 hour in the painting department. During the current production period, 240 hours of carpentry time are available and 100 hours in painting is available. Each table sold yields a profit of Rs. 7; each chair produced is sold for a Rs. 5 profit. Find the best combination of tables and chairs to manufacture in order to reach the maximum profit.

	Hours required to make 1 unit		
Department	Tables	Chairs	Available Hours
Carpentry	4	3	240
Painting	2	1	100
Profit	7	5	

Now, the mathematical formulation of the problem is as follows:

Maximize: P = 7x + 5y

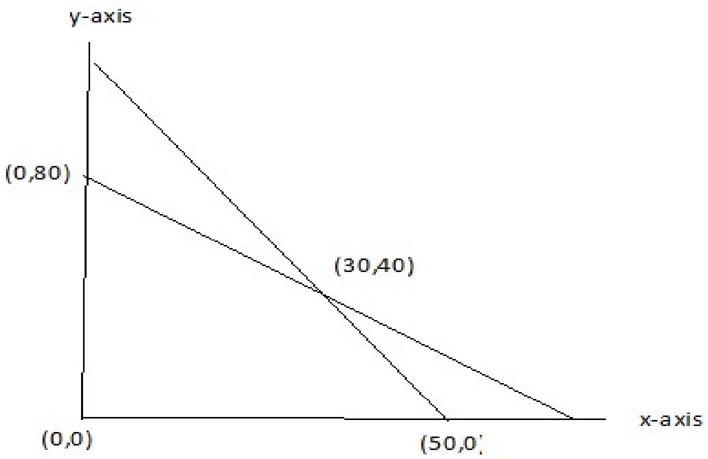
Subject to: 4x + 3y ≤ 240

2x +yY ≤ 100

x ≥ 0 ,y ≥ 0

where, x = number of tables to be produced &y = number of chairs to be produced

Thus, the graphical solution can be expressed as,



In this example the corner points are (0,0), (50,0),(30,40) and (0,80). Testing these cornerpoints on P = 7x+ 5y gives (As we know that the optimality occurs at the corner points of the feasible solution region.)

Corner Point	Profit (Rs.)
(0,0)	0
(50,0)	350
(30,40)	410
(0,80)	400

Because the point (30,40) produces the highest profit, so we conclude that producing 30tables and 40 chairs will yield a maximum profit of Rs. 410.

Solution through Simplex method

In practice, most of the problems contain more than two variables and are consequently too large problems to be tackled by conventional means. Therefore,an algebraic technique is used to solve large problems using Simplex Method. This method is carried out through iterative process systematically step by step, and finally the maximum or minimum values of the objective function is attained. The simplex method solves the linear programming problem in iterations to improve the value of the objective function. The simplex approach not only yields the optimal solution but also other valuable information to perform the analysis.

Note:

- The graphical method is one of the easiest way to solve a small Linear ProgrammingProblem. However, this is useful only when the decision variables are not more than two.
- It is not possible to plot the solution on a two dimensional graph when there are more than two variables and we must turn to more complex methods. Another limitation of graphical method is that, an incorrect or inconsistent graph will produce inaccurate answers, so one need to be very careful while drawing and plotting the graph. A very useful method of solving linear programming problems of any size is the so called Simplex method.

Discussion:

The linear programming technique helps to make the best possible use of available productive resources (such as time, labour, machines etc.).
In a production process, bottle necks may occur. For example, in a factory some machines may be in great demand while others may lie idle for some time.The quality of decision making is improved by this technique because the decisions are made objectively and not subjectively.

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Probability

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Introduction

Probability is the branch of mathematics concerning numerical descriptions of how likely an event is to occur, or how likely it is that a proposition is true. The probability of an event is a number between 0 and 1, where, roughly speaking, 0 indicates impossibility of the event and 1 indicates certainty. A simple example is the tossing of a fair (unbiased) coin. Since the coin is fair, the two outcomes ("heads" and "tails") are both equally probable; the probability of "heads" equals the probability of "tails"; and since no other outcomes are possible, the probability of either "heads" or "tails" is 1/2. Thus, Probability is a measure of the likelihood of an event to occur.

Properties

1. The probability of an event can be defined as the Number of favorable outcomes of an event divided by the total number of possible outcomes of an event.
2. Probability of a sure/certain event is 1.
3. Sum of probabilities of complementary events is 1.

4. Elementary event is an event that has only one outcome. These events are also called sample points. The Sum of probabilities of such elementary events of an experiment is always 1.
5. Probability of an event always lies between 0 and 1. It is always a positive number.

Applications

The measurement of the possibility of an event to occur is called probability. There are many applications associated with probability.

1. Application of Probability in Weather Forecast: Meteorologists collect the database related to weather and its changes worldwide by using different instruments and tools. The weather conditions for a particular hour, day, week, month and year can be predicted.

2. Application of Probability in Election Results: In many countries, elections play a vital role in politics. Political analysts use exit polls to measure the probability of winning or losing the candidate or parties in the elections. The probability technique is used to predict the results of voting after the election.

3. Application of Probability in Insurance: Insurance companies provide insurance policies or premiums based on the future forecast to the persons, vehicles etc. Insurance companies generally use theory of probability to frame any particular policy.

4. Application of Probability in Business: The marketing persons or salespersons promote the products to increase sales. The probability technique helps to forecast the business in future.

Discussion

Probability is simply how likely something is to happen. Whenever we are unsure about the outcome of an event, we can talk about the probabilities of certain outcomes- how likely they are. Probability theory is applied in everyday life in risk assessment and in trade on financial markets. Governments apply probabilistic methods in environmental regulation, where it is called pathway analysis.

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Probability Distribution Function

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Introduction

The probability Distribution function is the integral of the probability density function. This function is very useful because it tells us about the probability of an event that will occur in a given interval. A probability distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range. This range will be bounded between the minimum and maximum possible values, but precisely where the possible value is likely to be plotted on the probability distribution depends on a number of factors. These factors include the distribution's mean, standard deviation, skewness and kurtosis.

Definition

Let E be a random experiment, S be the corresponding event space, Δ be the class of events and define a mapping P : Δ → R . Then (S, Δ, P) is called the Probability space, X : S →R be a random variable and F : R → [0,1] is called the Distribution function of the random variable X (S,Δ,P). Thus, F(x) = P(-∞< X ≤x) ,x Є R.

Properties

Let F(x) be a probability distribution function corresponding to a random variable X . Then

(i) P (a< X ≤ b) = F(b)- F(a)

(ii) 0 ≤ F(x) ≤ 1 , for all x Є R

(iii) F(∞) =1 & F(-∞) =0

(iv) F(x) is a monotonically increasing function i.e. x1 ,x2 ЄR , with x1 < x2 ➔ F(x1) ≤ F(x2)

(v) F(x) is right continuous at some spectrum points.

(vi) F(x) may not be left continuous

Types of Probability Distribution

(i) Discrete: (a) Binomial (b) Poisson (c) Geometric

(ii) Continuous: (a) Uniform (b) Normal

(a) Binomial Distribution

Consider A discrete random variable X

Consider itsspectrum pointsas 0,1,2,…,n

P.M.F of X is given by $P(X=i)=f_i=\begin{cases} {}^nC_i p^i (1-p)^{n-i} & i=0,1,2....,n \\ 0 & elsewhere \end{cases}$

Where pЄ(0,1) and denoted by B(n,p)

Example: If a coin is tossed 5 times, find the probability of:

(a) Exactly 2 heads (b) At least 4 heads.

Solution:

(a) Number of trials: n=5. Probability of head: p= ½. probability of tail: q (1-p) =1/2. For exactly two heads: i=2 and P(X=2) = ${}^{5}C_{2}p^{2}(1-p)^{5-2} = \frac{5!}{2!3!}(\frac{1}{2})^{2}(\frac{1}{2})^{3}$. So, $P(X=2)=\frac{5}{16}$

(b) For at least four heads, X ≥ 4, P(X ≥ 4) = P(X = 4) + P(X=5).

Hence, P(X = 4) = ${}^{5}C_{4}p^{4}(1-p)^{5-4} = \frac{5!}{4!1!}(\frac{1}{2})^{4}(\frac{1}{2})^{5-4} = \frac{5}{32}$. P(X = 5) = ${}^{5}C_{5}p^{5}(1-p)^{5-5} = \frac{1}{32}$. Therefore, P(X ≥ 4) = $\frac{5}{32}+\frac{1}{32}=\frac{3}{16}$.

(b) Poisson Distribution: It is a discrete distribution whose spectrum is given by $X_i = i$ and its p.d.f. , f_i at $X = X_i$ is given by $f_i = e^{-\mu}\mu^{i}$, i=0,1,2,…….∞ where μ (>0) is the only parameter of the distribution.

Condition of Poisson distribution:

(i) Events are discrete (ii) Events cannot happen at the same time (iii) Events are independent

Example:

Let us consider k= 2 deaths by horse kickand λ= 0.61 deaths by horse kick per year. e = 2.718

Now, P(X = k) = $\frac{e^{-\lambda}\lambda^{k}}{k!}$ =0.101

(c) Geometric Distribution:

It is a discrete distribution having spectrum points 0 ,1,2…..where

p.d.f is given by $f_i = P(X=i) = \begin{cases} p(1-p)^{i} & i=0,1,2....\infty \\ 0 & elsewhere \end{cases}$, ,where 0<p<1

Geometric distribution (a discrete probability distribution) is used (i) when there are only 2 outcomes (success or failure) (ii) repeated until a successful outcome occurs&success occurs on the nth trial (iii) tells the probability of when the success is likely to occur.

Formula: $P(n) = p(1-p)^{n-1}$

Example:

What is the probability of getting the first 6 on the fourth roll of a die?

Solution: we have $P(n) = p(1-p)^{n-1}$. Here p= $p = \frac{1}{6}$ and so, $P(4) = 0.096$.

(d) Uniform Probability Distribution:

Over an interval (a,b), $f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & , elsewhere \end{cases}$

It is denoted by $U(a.b)$.

Example: A circuit board failure causes a shutdown of a computing system until a new board is delivered. The delivery time X is uniformly distributed between 1 and 5 days. What is the probability that it will take 2 or more days for the circuit board to be delivered?

Solution: $f(x;1,5) = \frac{1}{5-1} = \frac{1}{4}$. So, $P(X \geq 2) = \int_2^5 \frac{1}{4} dx$ =0.75 .

(e) Normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$, -∞<x<∞.

It is denoted by N(m,Ϭ) variate.

▪ The distribution is symmetric and its skewnessmeasurement is zero.

▪The entire family of normal probability distributions is defined by its mean μ and its standard deviation Ϭ.

▪The highest point on the normal curve is at the mean , which is also the median and mode

Discussion:

Probability distribution indicates how probabilities are allocated over the distinct values for an unexpected variable. A probability distribution has various belongings like predicted value and variance which can be calculated. Even when all the values of an unexpected variable are aligned on the graph, then the value of probabilities yields a shape.

References:

[1] Arup Mukherjee, Fundamental Treatise on Probability and Statistics

[2] https://www.cuemath.com/data/probability-distribution/

Theory of Equation – Solution of Cubic and Biquadratic Equations

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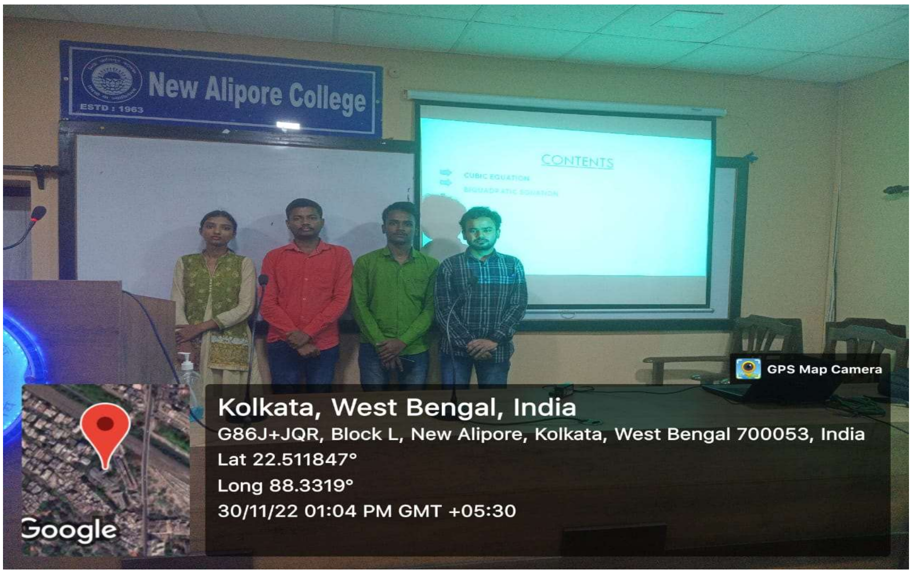

Introduction

Every equation of nth degree has a total n real or complex roots. If α is the root of a equation $f(x)=0$, then the polynomial $f(x)$ is exactly divisible by $(x-\alpha)$ i.e., $(x-\alpha)$ is the factor of the given polynomial $f(x)$. In algebra, the study of algebraic equations is called the theory of equations. A polynomial is an expression consisting of one or more terms. A main difficulty of the theory of equations was to know when an algebraic equation has an algebraic solution. In this article, we will discuss on the solutions of cubic and biquadratic equations.

- **Solution of cubic equation (By Cardan's Method)**

Let f(x)=0 be a cubic equation. Where $f(x) = ax^3 + 3bx^2 + 3cx + d = 0$ and a≠0.

Now reduce the equation to standard form of Carden's by the transformation z = ax+b and it reduces to $z^3 + 3Hz + G = 0$,where $H = ac - b^2$ $G = a^2 d - 3abc + 2b^3$.

Now, to solve the equation let us assume $z = u + v$

Then, $z^3 = u^3 + v^3 + 3uv(u+v)$. Or, $z^3 - 3uv(u+v) + (u^3 + v^3) = 0$

Now comparing with $z^3 + 3Hz + G = 0$, we have $u^3v^3 = -H^3$ and $(u^3+v^3) = -G$.

Thus, u^3 & v^3 are the roots of the equation, $t^2+Gt-H^3=0$

Solving the cubic equation, we get its two roots and let us suppose,

$$u^3=(1/2)\{-G+\sqrt{(G^2+4H^3)}\} \quad \text{and} \quad v^3=(1/2)\{-G-\sqrt{(G^2+4H^3)}\}$$

If p denotes any one of the three values of $\sqrt[3]{[(1/2)\{-g+\sqrt{(G^2+4H^3)}\}]}$, then the values of u are p ,$p\omega$, $p\omega^2$ where ω *is imaginary cube* root of unity . Since uv = -h the three corresponding values of v are (–H/p) , $(-\omega^2 H/P)$, $(-\omega H/P)$. Hence the values of z are {p-(h/p)},

$\{\omega\, p-(\omega^2 H/P)\}$, $\{\omega^2 P-(\omega H/P)\}$. So the three values of x are [(1/a){p-(H/p)-b}] ,

$[(1/a)\{\omega\, p-(\omega^2 H/P)\}-b]$, $[(1/a)\{\omega^2 P-(\omega H/P)\}\ -b\}]$. This gives a complete solution of the given equation and the method of solution is called the cardan's method.

Example:

Solve the equation $x^3 -15x^2-33x+847= 0$.

Solution:

Let us apply the transformation x = y+h in order to remove the second term. Now, the transformed equation is $(y+h)^3 - 15(y+h)^2 - 33(y+h) + 847 = 0$.

So, h=5 and the equation reduces to , $y^3-108y+432 = 0$

let, y = u+v. Then , $y^3 = u^3 +v^3 + 3uv(u+v)$ or, $y^3-(u^3 + v^3) - 3uvy = 0$

Now, comparing with the equation , $y^3-108y+432 = 0$ we get, uv = 36 and $u^3 +v^3 = -432$.

Then the three values of u are -6 , -6ω, $-6\omega^2$ *and* the corresponding values of

v are -6 , $-6\omega^2$, -6ω.

Then y = -12 ,6, 6 and the roots of the given equation are -7,11,11.

- **Solution of Bi-quadratic Equation (By Ferrari's Method)** let f(x)= 0 be a bi-quadratic equation of the form $ax^4+4bx^3+6cx^2+4dx+e=0$ such that $a\neq 0$.

Now, multiplying by a, $a^2x^4+4abx^3+6acx^2+4adx+ae = 0$

let the left hand expression be expressed as the difference of the two squared terms in the form

$(ax^2+2bx+\lambda)^2 - (mx + n)^2$

Now, comparing with the left hand expression of the above equation, we have

$6ac = 4b^2+2\lambda a-m^2$, $4ad = 4b\lambda -2mn$ & $ae = \lambda^2 - m^2$

Eliminating m , n from these we have $4(b\lambda-ad)^2 = (2\lambda a+4b^2-6ac)(\lambda^2-ac)$

This is a cubic equation in λ which gives at least one real root λ.

Thus, corresponding to real value of $\lambda = \lambda_1$ we have the values of m & n by using the relation $mn=2b\lambda_1-2ad$.

Now put in the form $(ax^2+2bx+\lambda_1)^2-(m_1x+n_1)^2=0$, where m_1 , n_1 are the values of m , n corresponding to λ_1 . The roots of the quadratic equations $[(ax^2+2bx+\lambda_1)\pm(m_1n+n_1)] = 0$

given the above solution of the given bi-quadratic equation.

Example:

Solve: $x^4+6x^2+14x^2+22x+5=0$

Solution:

Let $x^4+6x^2+14x^2+22x+5= (x^2+3x+\lambda)^2-(ax+b)^2$ where a, b and λ are constants.

Now, equating coefficient of like powers of x, we have

$14 = 9+2\lambda-a^2$, $or,\ a^2 = 2\lambda - 5$, $22 = 6\lambda-2ab$, $or\ ab = 3\lambda - 11$, $5 =\lambda^2-b^2$

$$or,\ b^2 = \lambda^2 - 5$$

Eliminating a , b we have $(\lambda^2-5)(2\lambda-5)-(3\lambda-11)^2=0$

$$\text{or}, \ 2\lambda^3 - 14\lambda^2 + 56\lambda - 96 = 0$$

$$\text{or}, \ (\lambda - 3)(2\lambda^2 - 8\lambda + 32) = 0$$

$$\text{Therefore } \lambda = 3, \ 2 \pm 2\sqrt{3}i.$$

Now, taking $\lambda = 3$, we have the roots of the given equation as $-2 \pm \sqrt{3}$, $-1 \pm 2i$

Discussion

The methods as discussed in this article will help to get an exact solution of a cubic or a bi-quadratic equation.

Reference:

[1] S. K. Mapa, Higher Algebra: Classical, Sarat Book House.


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